Please use our materials!

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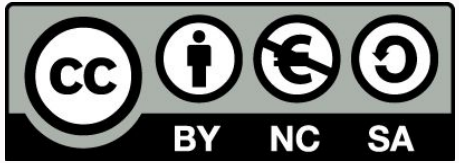
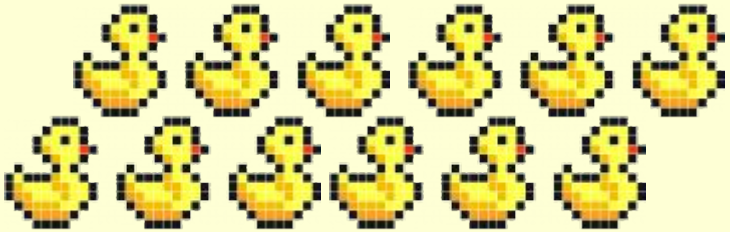
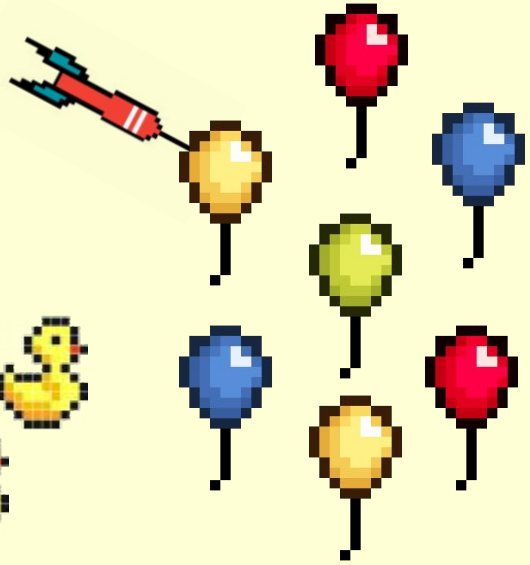
We also humbly request that you email sarah.adams@olin.edu if you use these materials, as we are tracking their impact and how far they travel!







CARNIVAL GAMES

HELLO!

Junwon

(he/him)

Reid

(they/them)

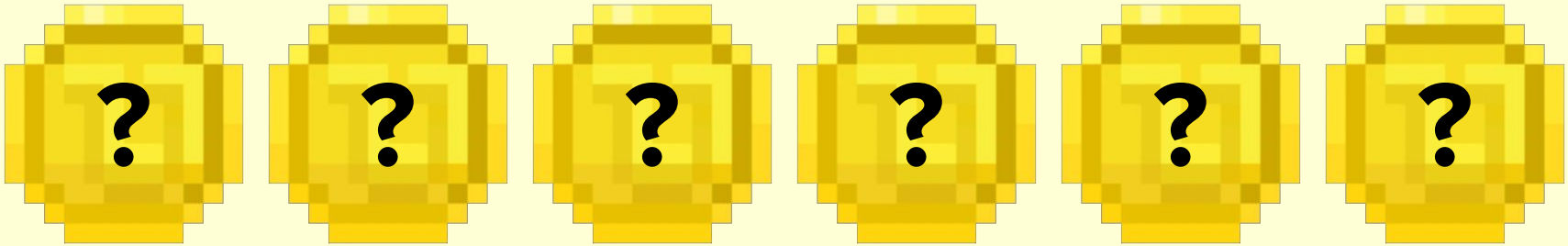
Ashley

(she/her)

FLIPPING A COIN

If we have a normal coin, and we flip it 6 times, how many times would you expect to get heads (instead of tails)?

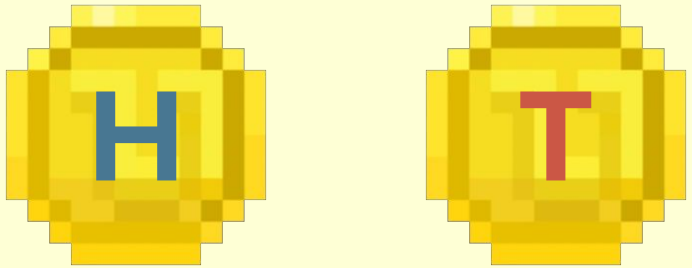
Type it in the chat!



NOW, FLIP A COIN SIX TIMES

Write the total number of HEADS you get in the chat when you're done

Don't have a coin? You can ask Siri or head to justflipacoin.com/



SO WHAT HAPPENED?

If everything was even and fair, you'd probably expect to get 3 heads and 3 tails... so why didn't everyone get that?

PROBABILITY

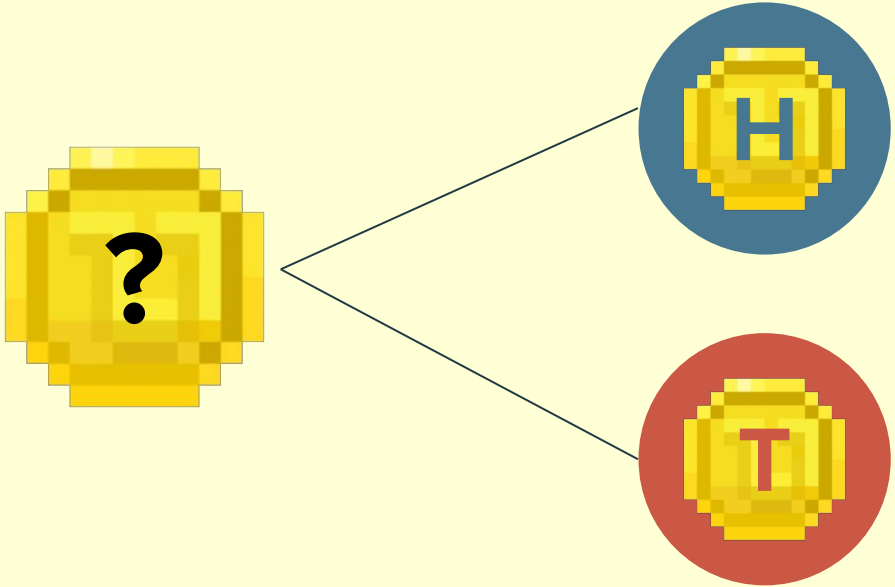
Probability is a *prediction*, not a *promise!*

LETS PLAY A (very basic) CARNIVAL GAME!

TO WIN: FLIP A COIN ONCE, GET HEADS

Go ahead and flip! Did you win?

What was the *probability* you were going to win? (We can call this P(winning) or P(heads))



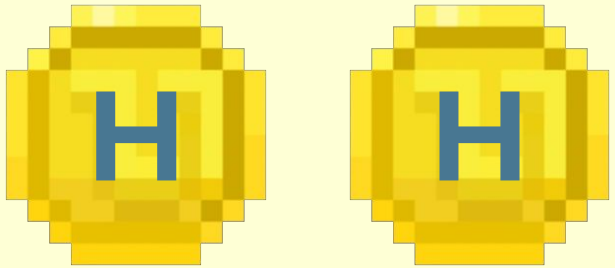
$$\frac{1}{2}$$

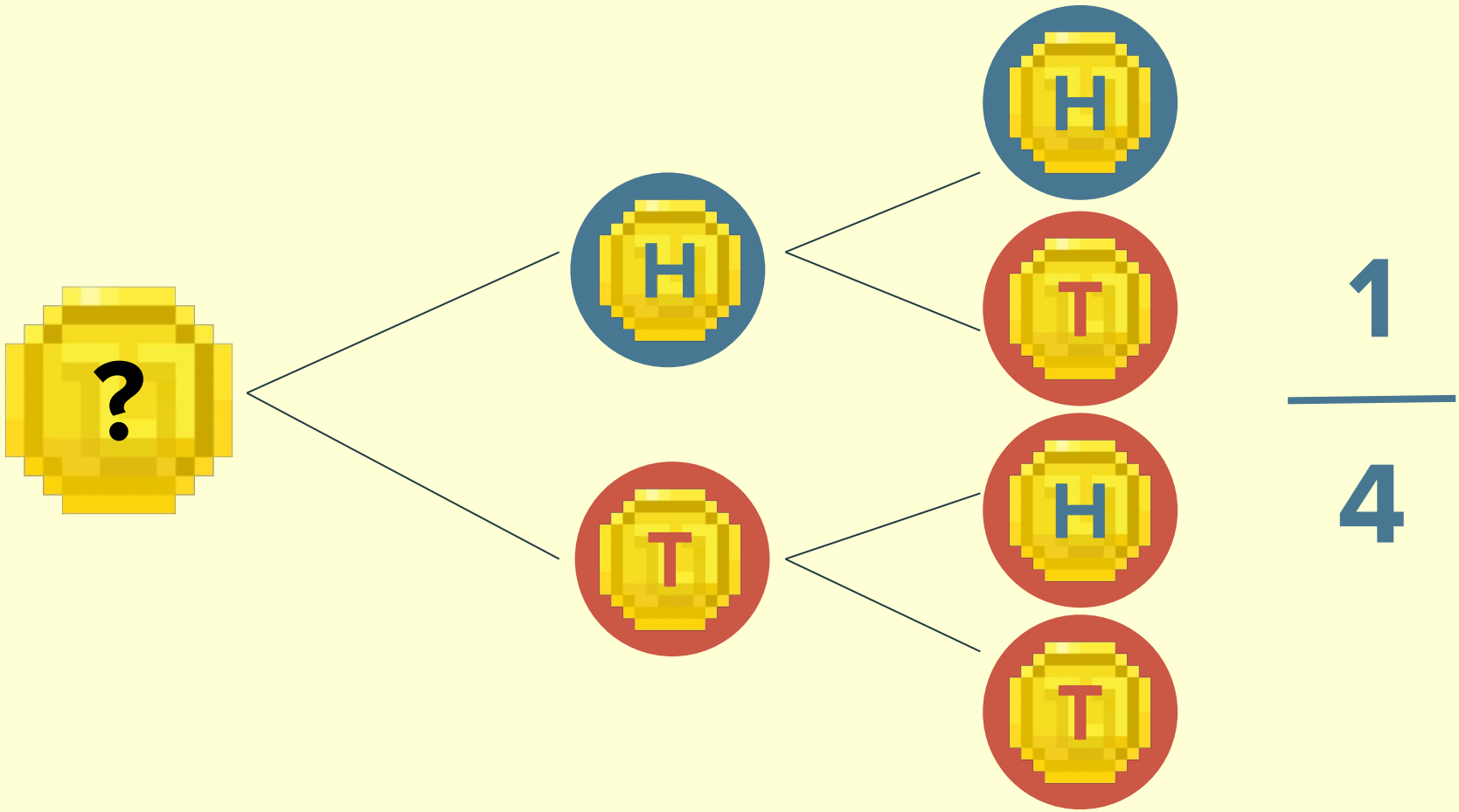
TO WIN: FLIP TWICE, GET HEADS *BOTH TIMES*

Go ahead and flip two times! Did you win the game this time?

What was the probability you were going to win this time?

(In other words, what is *P(two heads in a row)* ?)

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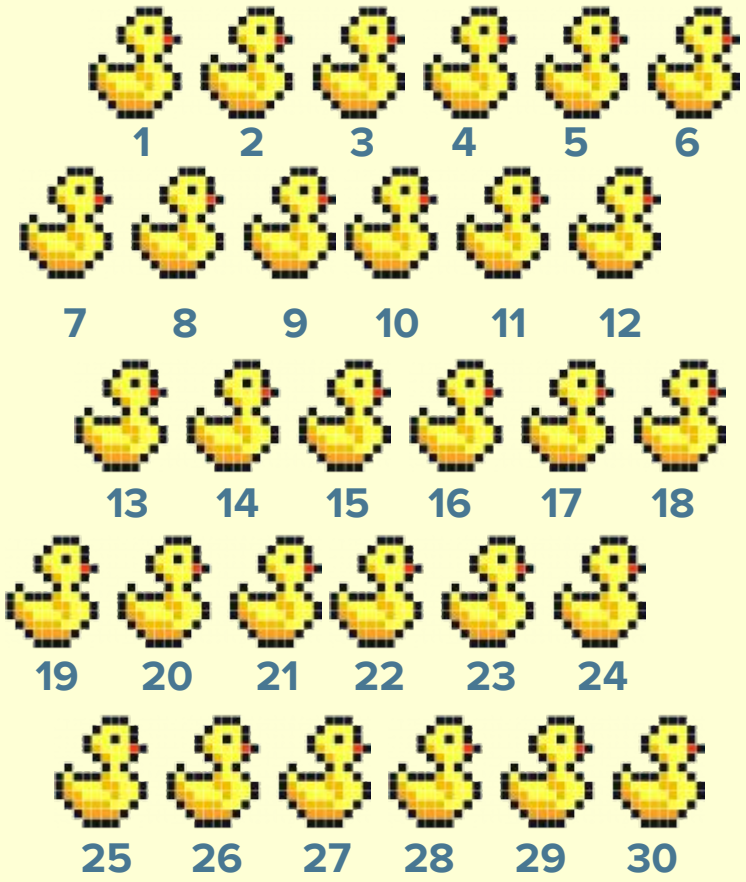


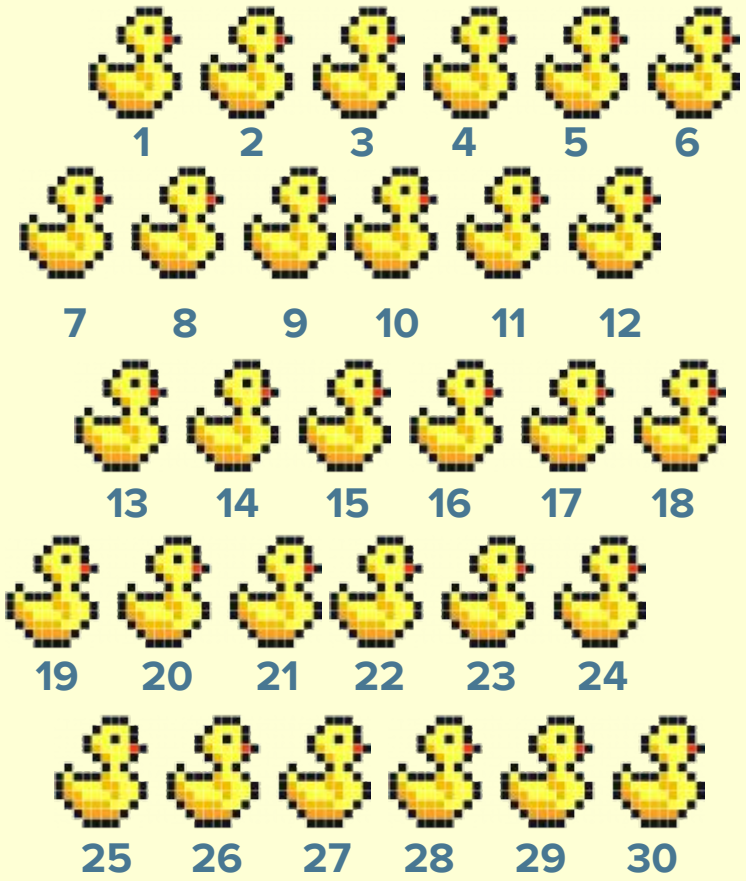
NOW LET'S PLAY A DIFFERENT GAME

To play this game, you need to pick a duck from a pool

Every duck has a number on it!

Here's **30 ducks.** They are numbered 1-30

PICK A DUCK!



Let's say to win, you need to pick a duck with an odd number on it.

What is
P(picking an odd numbered duck)?

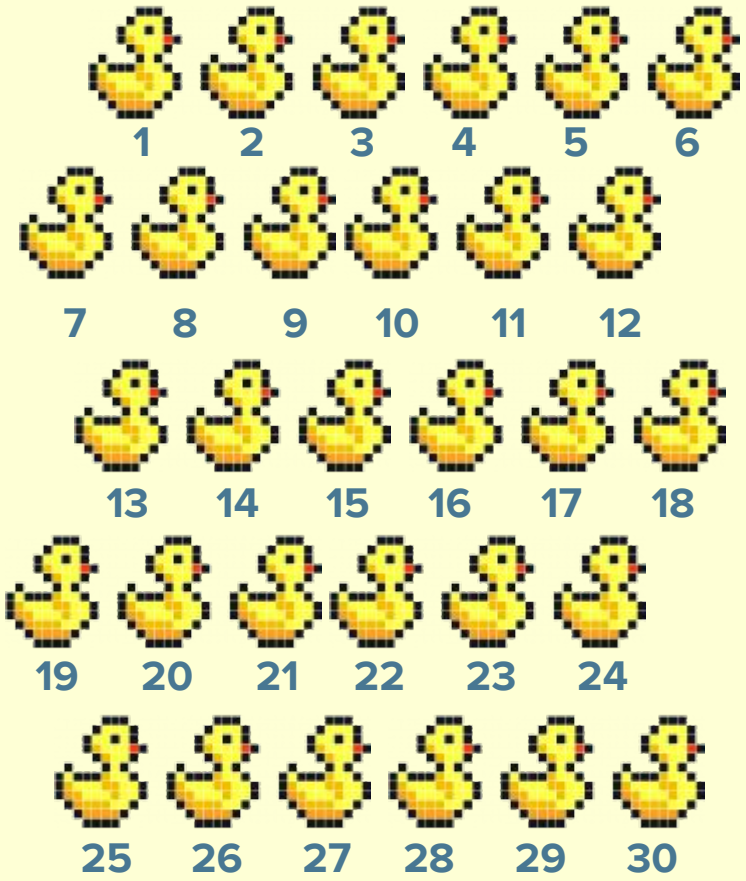
SOLUTION

There are 15 odd numbered ducks!

That's 15/30, which we can simplify to

½, which is

A **50% chance of winning!**



CHANGING THE ODDS

Now, let's say that you have to pick a duck with a multiple of 5 on it to win.

What is
P(picking a multiple of 5)?

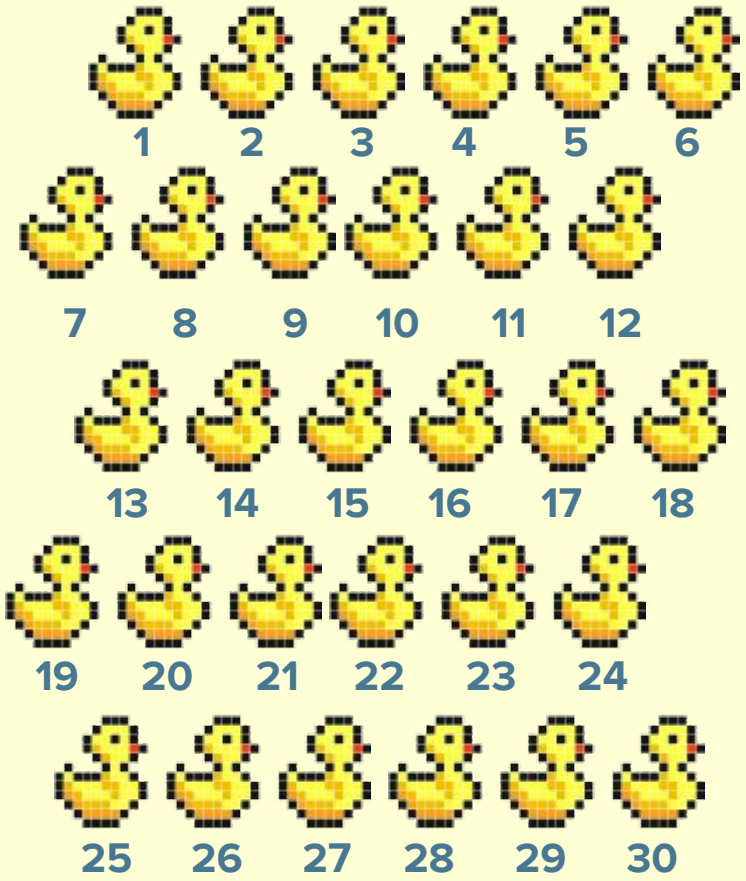
SOLUTION

There are 6 ducks that are multiples of 5

That's 6/30 which simplifies to 1/5

So you have a **20% chance of winning**

PICK A DUCK!



Alright, now we need to pick 1 duck with odd multiple of 5 on it to win.

What is
P(picking an odd multiple of 5)?

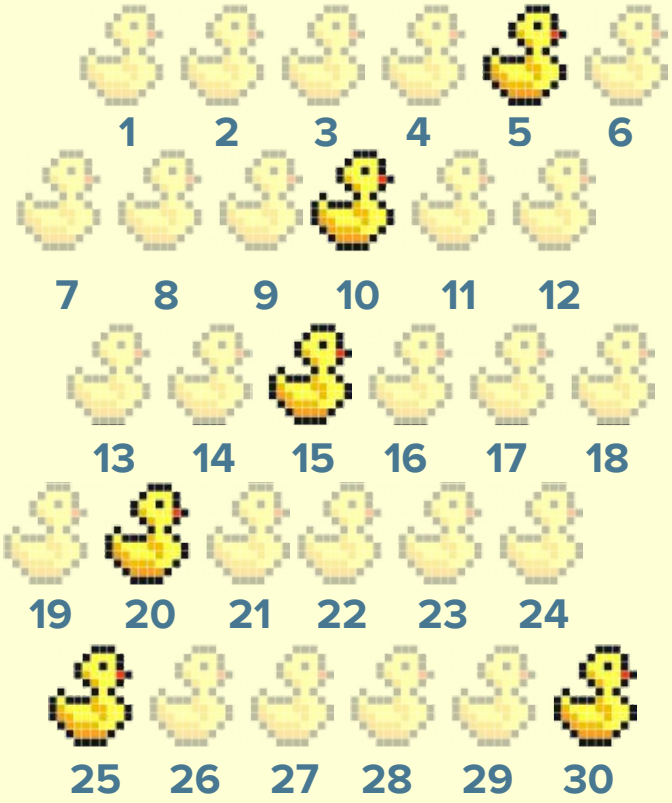
SOLUTION

There are 3 ducks that are odd multiples of 5 (5, 15, 25).

That's 3/30 which simplifies to 1/10

So you have a **10% chance of winning!**

P(Odd Duck) = 1/2



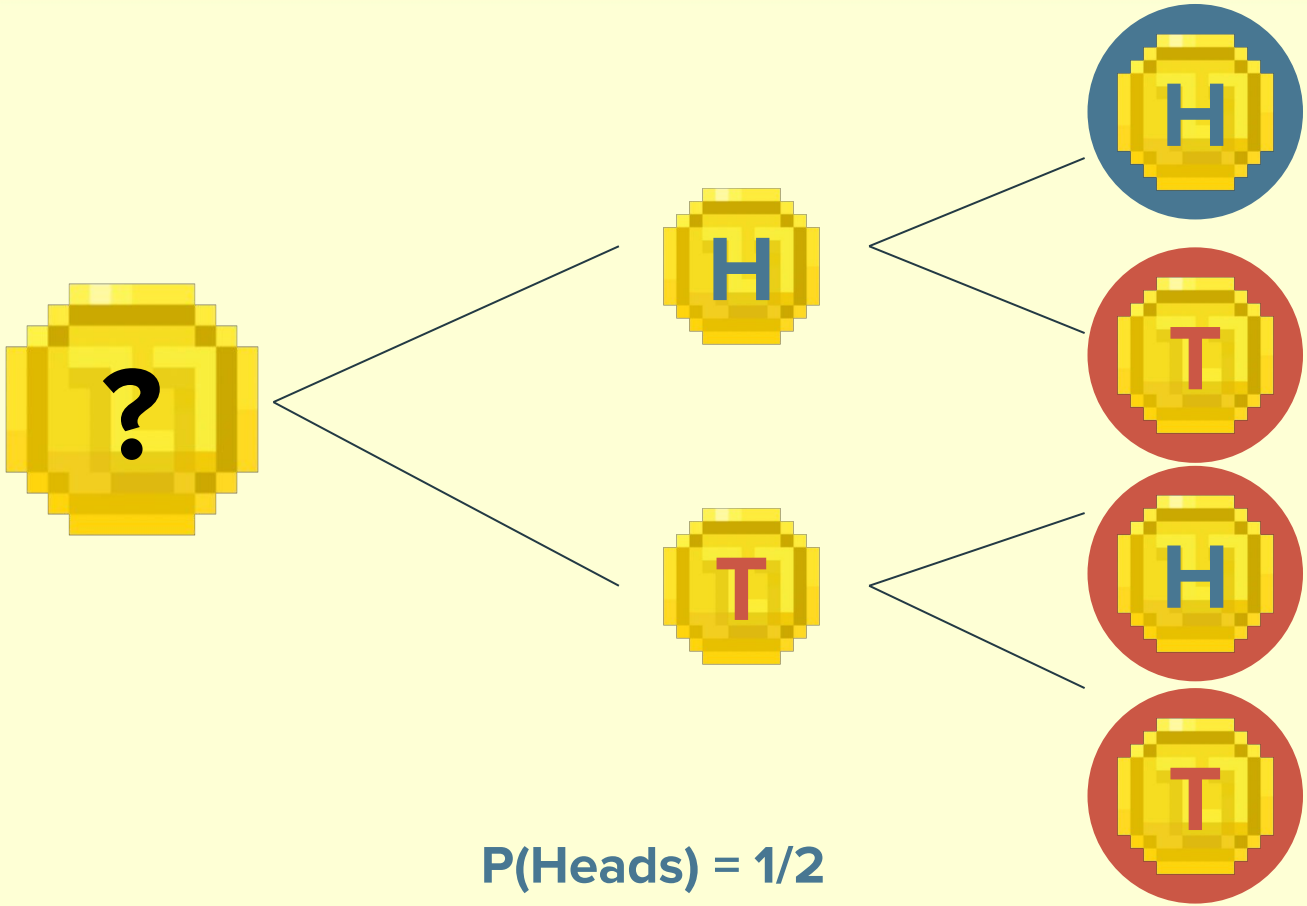
P(Multiple of 5 Duck) = 1/5

P(Odd Duck) = 1/2

P(Multiple of 5 Duck) = 1/5

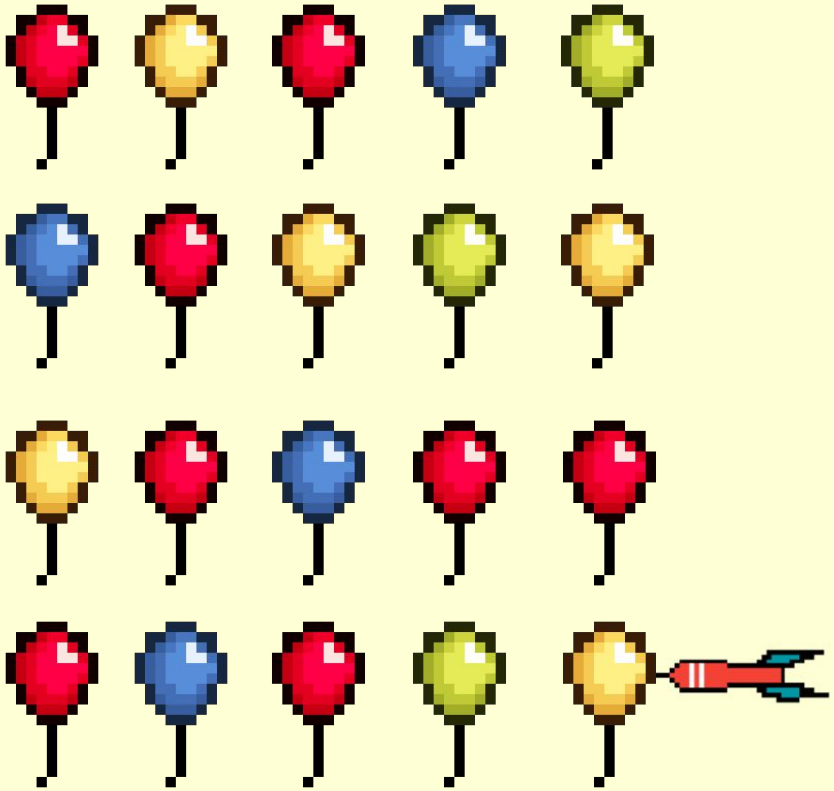
$$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

P(Odd AND Multiple of 5 Duck) = 1/10



P(Heads) = 1/2

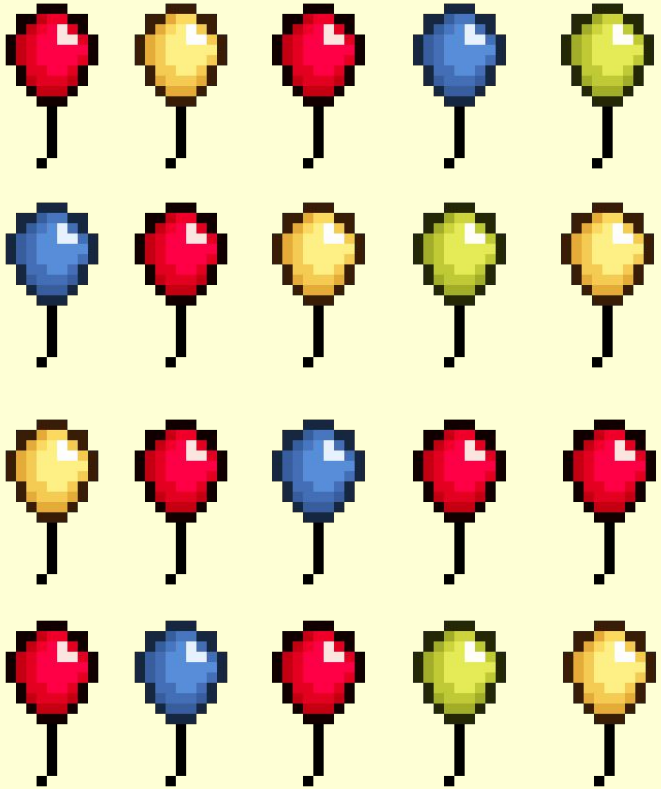
P(Heads) X P(Heads) = ½ X ½ = P(Heads AND Heads) = 1/4



Let's try another game!

In this game, we need to pop a balloon with a dart.

We'll assume that we aren't very good at aiming, so we aren't any more likely to hit one balloon over another. But eventually we *will* hit a balloon.



First Win Condition

Let’s say to win this game, I need to pop a red balloon.

What’s the chances I’ll win?

How many red balloons do we have?

How many balloons total?

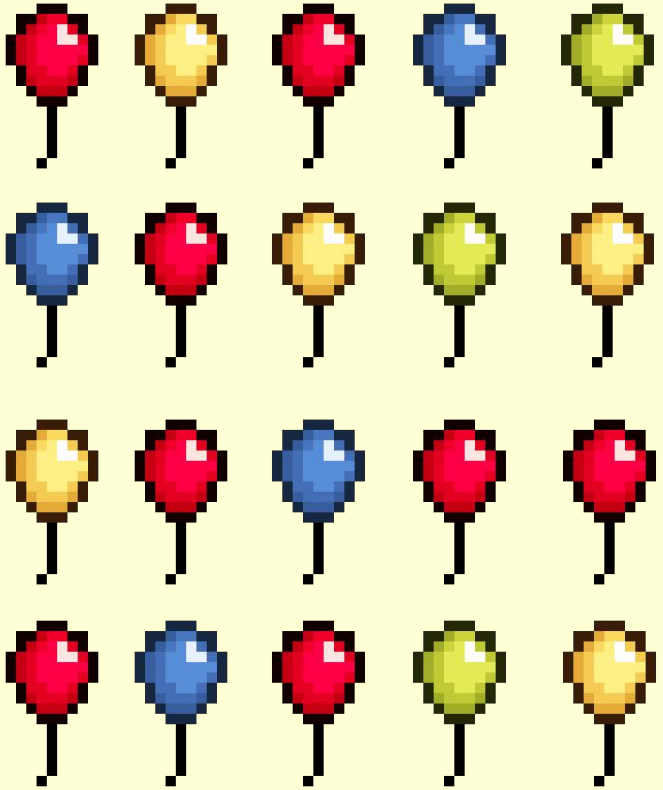
First Win Condition

8 Red Balloons out of **20 Total Balloons** is an 8/20, or 2/5 chance of winning.

If we divide that out, we have 0.40, or a **40% chance** of winning!

First Win Condition

40% seems like a pretty good chance of winning. But what if the rule to win was different? Would we still want to play?

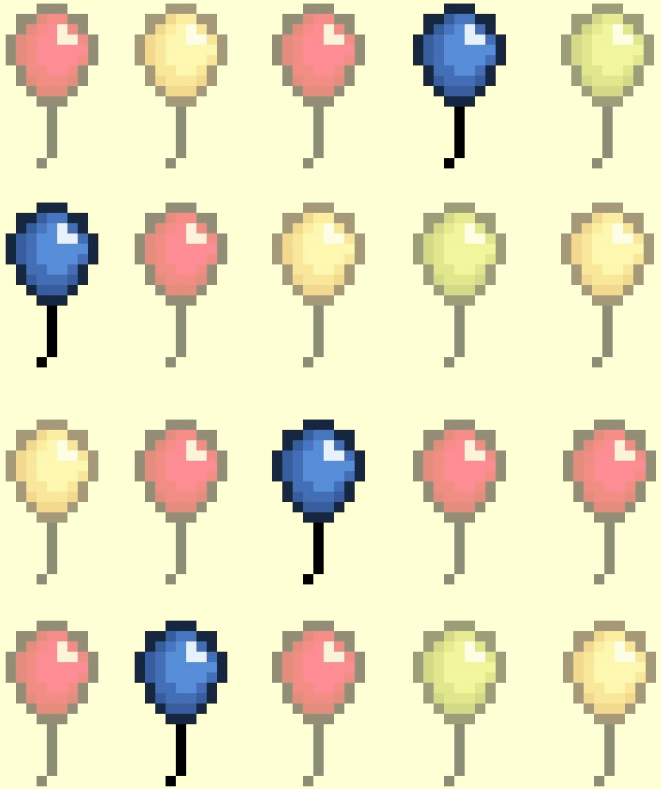


Another Win Condition

Now let's say we need to hit a **blue** balloon to win.

What's the chances I'll win, or P(blue)?

Pop it in the chat!

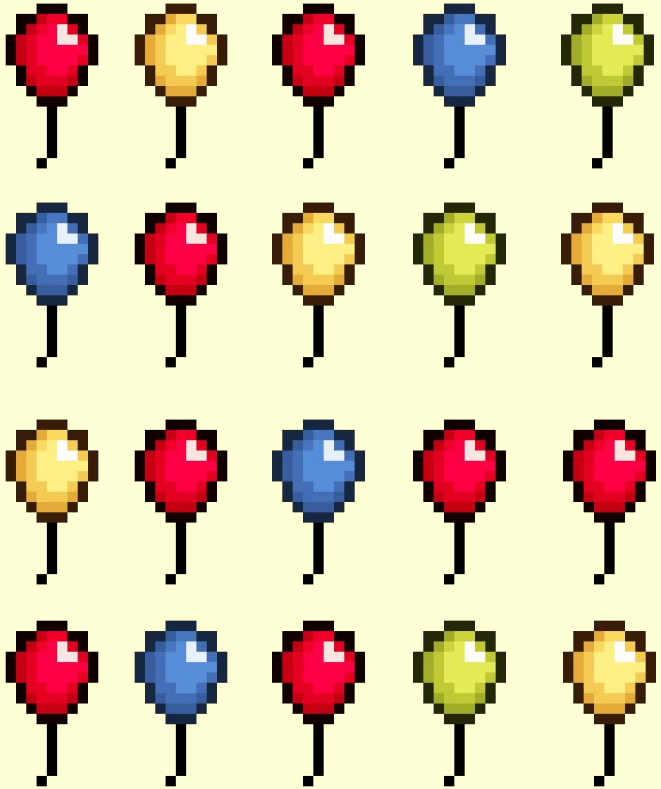


Another Win Condition

4 blue balloons out of
20 total balloons is
4/20 *or* 1/5

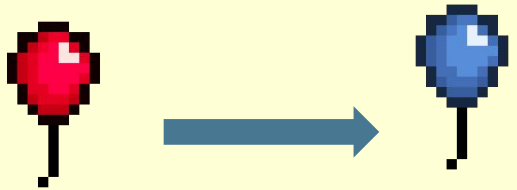
(You'll notice the denominator, 20, stays the same! We have the same number of balloons in our game.)

1 divided by 5 is .2, which gives us a **20% chance** of winning this game!

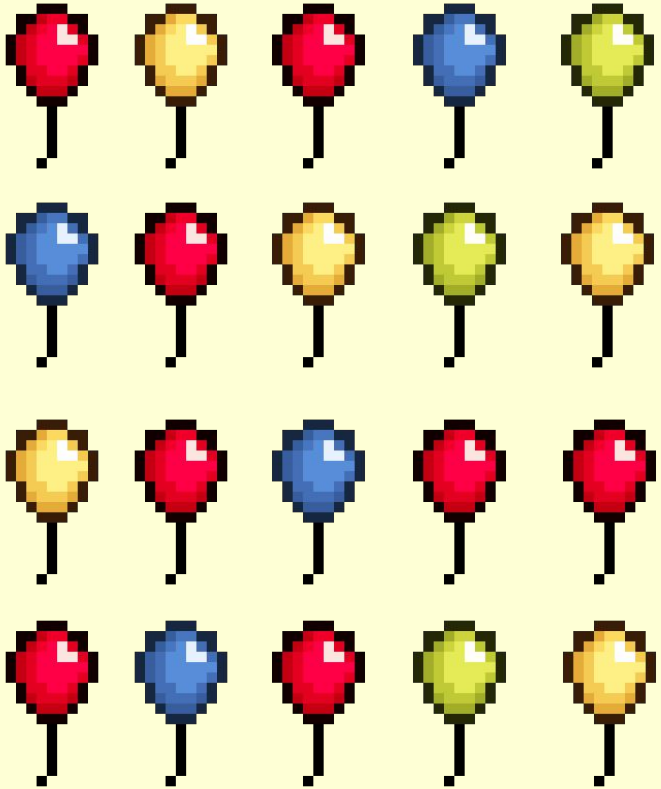


Popping TWICE

This new game gives out two prizes: a small candy for popping a red balloon, but a HUGE teddy bear for anyone who can hit a **red balloon and then a blue balloon.**



1st 2nd



Popping TWICE

We know P(red balloon first),
but we don't know
P(blue balloon after popping red)

P(red balloon first) was **40%**

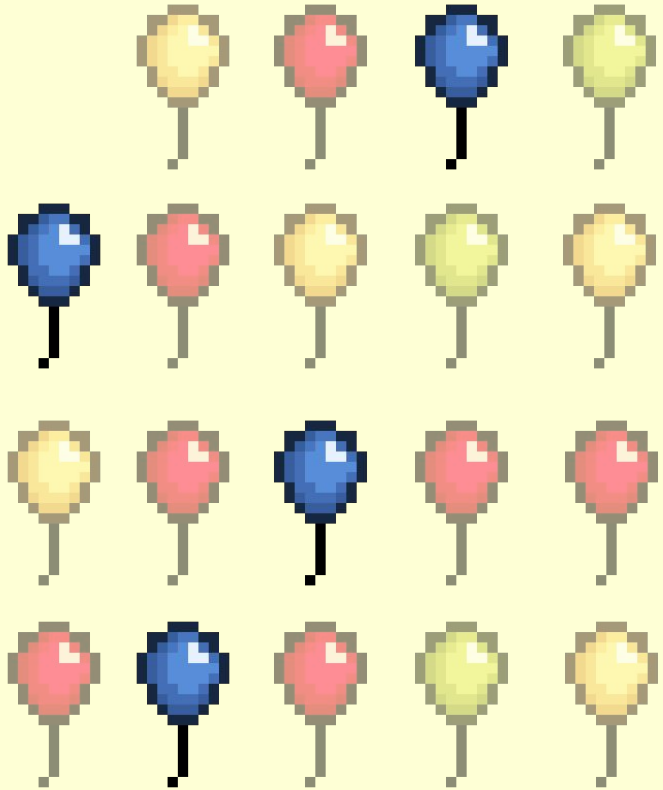
A QUICK NOTE ON NOTATION

P(event) = “The probability of event happening”

P(event | condition) = “The probability of event happening, GIVEN that we know ‘condition’ is true”

P(popping blue) = “The probability of popping a blue balloon”

P(popping blue | a non-blue popped) = “The probability of popping a blue balloon, given that we already popped a non-blue balloon.”



Popping TWICE

How many blue balloons are there after we've popped a red balloon?

Still 4!

How many balloons are there, total?

Only 19!

So, P(popping blue | popped one red) is **4/19,** or about **21%**

Popping TWICE

1st 2nd

$$\frac{8}{20} \times \frac{4}{19} = \frac{32}{380} = 8.4\%$$

Popping TWICE

1st 2nd

P(red) × P(blue | red) = P(red AND blue)

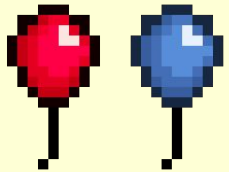
$$\frac{8}{20} \times \frac{4}{19} = \frac{32}{380} = 8.4\%$$

What was different between the coins and the balloons?

INDEPENDENT event - the first coin toss doesn't affect the second!

P(Heads) x P(Heads) = P(Heads AND Heads)

DEPENDENT event - popping the first balloon changes the probability of the next throw!



P(red) x P(blue | red) = P(red AND blue)

MAKE YOUR OWN GAME

Now, can you invent your own game?

If you want, you can use coins, balloons, ducks, or something else! Go ahead and draw it out on paper, write about it in the chat, or hop on the voice chat and talk it out!

Try and set it so you have a P(winning) of 10-25%. Or can you make two prizes, one harder to win than the other?

THANK YOU!!!